From part 1, the CSP program would reach about size 23 before maxing out the 2 second limit. The first improvement I tried was to randomize the row that the program will process next. I did this simply by creating a list of rows still unoccupied (-1) and randomly selected from the list. My reasoning for this change in my row selection stemmed from our previous Informed Search Lab 15 Puzzle. In 15 Puzzle, A\* values with a multiplier meant that we could introduce variability into our output. This gives us a wider range of returned solutions. When applied to CSP, variability means a chance of finding a faster solution with less backtracking, therefore improving run-time. This theory was proved true when CSP would be able to reach about size 35-40 before maxing out the 2 second limit.

Randomizing the row selection will increase the chance of picking a faster path, however the results are inconsistent. In order to see why some runs would result in significantly more solutions than others, ran the pycharm profiler on the program. This returned a chart of the number of times it would call the get\_sorted\_values or get\_random\_var methods. Below is a chart of 5 profiles:

|  |  |
| --- | --- |
| Puzzle Size Limit | get\_sorted\_values Call Count |
| 29 | 696975 |
| 35 | 307605 |
| 39 | 382262 |
| 43 | 329482 |
| 56 | 294553 |

The chart shows that the when the program displays a 2 second termination at a smaller size, the number of calls is much greater than at a greater size. If each method is called exactly once at every level of the recursion/backtracking, I reasoned that the biggest reason for time consumption for a puzzle is the frequency and depth of backtracking. If I found a way to minimize the amount of backtracking, then I should be able to fit a greater sized puzzle within the 2 second limit. I implemented a method that would find the most constrained row, the row that had the least number of value options, instead of random. My thinking was that by choosing the most constrained, the program would as always choose to run through the shallow depth branches first, processing through more complete branches faster. Processing through more branches faster improves the chance of finding the target branch. This method improved my 2 second time limit to a puzzle size of 75.

Another change that resulted from the pycharm profiler results was in the implementation of the method, get\_sorted\_values. I observed that the get\_sorted\_values took more than 90% of the runtime, and obviously if a method called at each recursion level were to take a long time, the effects would be multiplied by the backtracking. I was able to change the efficiency from O(n^2) to O(n). Although the change did not improve the size able to be completed under 2 seconds, it did lower the average amount of time of every puzzle by about 0.02 seconds.

The final progress I made with the puzzle size was when I combined the concept of variability and the get\_most\_constrained methods. To do this, I kept choosing the most constrained row, but randomly chose the available columns to put into the row. Because get\_sorted\_values returns a list of the possible columns, I used random.shuffle(). From this combination, as always with choosing the next step randomly, there was variability with the results. However, the variability was able to improve my runs significantly when it chanced upon the right path. Currently, my program varies from being able to solve puzzles of about size 70 to the highest observed size 139. The results of 139 is displayed below:

[0, 4, 7, 5, 2, 6, 1, 3]

For size 8, the time was 0.00038505700000000115.

[6, 8, 3, 1, 4, 7, 5, 0, 2]

For size 9, the time was 0.00020310700000000098.

[0, 2, 5, 7, 9, 4, 8, 1, 3, 6]

For size 10, the time was 0.0038170709999999983.

[5, 7, 2, 6, 3, 9, 0, 8, 10, 4, 1]

For size 11, the time was 0.0007387309999999994.

[5, 3, 8, 11, 2, 4, 7, 10, 0, 9, 6, 1]

For size 12, the time was 0.0009838.

[7, 3, 1, 6, 9, 0, 2, 4, 12, 10, 8, 11, 5]

For size 13, the time was 0.0006530460000000043.

[12, 3, 7, 11, 4, 1, 5, 0, 9, 6, 13, 2, 10, 8]

For size 14, the time was 0.00045663800000000226.

[13, 9, 0, 12, 8, 2, 5, 10, 1, 14, 4, 11, 7, 3, 6]

For size 15, the time was 0.0009446590000000005.

[6, 11, 13, 8, 1, 9, 7, 3, 15, 0, 10, 5, 14, 12, 4, 2]

For size 16, the time was 0.0006248360000000036.

[9, 5, 3, 16, 0, 12, 6, 15, 10, 4, 1, 14, 11, 2, 7, 13, 8]

For size 17, the time was 0.0011378920000000015.

[15, 1, 16, 0, 9, 14, 10, 2, 4, 11, 13, 3, 5, 12, 8, 6, 17, 7]

For size 18, the time was 0.002721139999999997.

[0, 9, 18, 2, 11, 7, 3, 12, 14, 4, 13, 5, 16, 1, 10, 6, 17, 15, 8]

For size 19, the time was 0.004009599000000003.

[15, 3, 12, 9, 7, 1, 18, 11, 0, 14, 10, 2, 19, 8, 13, 4, 17, 5, 16, 6]

For size 20, the time was 0.0031661409999999973.

[15, 8, 20, 9, 3, 1, 7, 17, 12, 18, 13, 19, 2, 5, 14, 10, 0, 4, 11, 16, 6]

For size 21, the time was 0.0024933499999999983.

[5, 17, 0, 4, 21, 8, 2, 9, 16, 18, 10, 1, 7, 19, 3, 14, 20, 11, 15, 12, 6, 13]

For size 22, the time was 0.006676084000000013.

[19, 16, 11, 1, 8, 5, 0, 2, 18, 20, 12, 3, 9, 14, 4, 21, 7, 22, 17, 15, 13, 10, 6]

For size 23, the time was 0.0031125429999999954.

[1, 16, 22, 9, 7, 21, 8, 0, 20, 14, 10, 19, 6, 12, 2, 4, 13, 15, 3, 23, 11, 17, 5, 18]

For size 24, the time was 0.0030215690000000017.

[5, 11, 24, 16, 3, 9, 22, 2, 19, 12, 1, 18, 4, 21, 8, 15, 17, 23, 20, 6, 0, 14, 10, 13, 7]

For size 25, the time was 0.011457207999999997.

[6, 4, 10, 14, 19, 3, 1, 23, 25, 16, 9, 15, 24, 18, 0, 12, 5, 7, 20, 13, 2, 8, 22, 11, 17, 21]

For size 26, the time was 0.0028276299999999976.

[14, 26, 6, 15, 11, 23, 1, 3, 25, 2, 16, 13, 9, 4, 22, 24, 21, 5, 7, 19, 0, 20, 12, 17, 8, 10, 18]

For size 27, the time was 0.007517425999999994.

[11, 7, 3, 25, 12, 15, 20, 22, 27, 16, 26, 1, 6, 9, 17, 0, 18, 10, 23, 2, 4, 19, 8, 14, 24, 13, 21, 5]

For size 28, the time was 0.003184124999999996.

[1, 18, 15, 11, 6, 8, 16, 2, 28, 24, 10, 27, 0, 3, 12, 22, 5, 14, 23, 25, 19, 13, 26, 9, 4, 21, 17, 20, 7]

For size 29, the time was 0.011818992.

[13, 8, 14, 0, 27, 10, 21, 18, 25, 17, 9, 7, 22, 11, 6, 29, 16, 4, 24, 3, 15, 2, 23, 26, 12, 5, 28, 20, 1, 19]

For size 30, the time was 0.005533606999999996.

[8, 2, 27, 16, 22, 20, 28, 7, 19, 3, 12, 30, 4, 18, 9, 5, 1, 23, 6, 29, 13, 17, 21, 26, 11, 14, 24, 10, 25, 15, 0]

For size 31, the time was 0.003605149000000002.

[25, 23, 20, 31, 17, 7, 2, 0, 29, 18, 26, 12, 19, 13, 1, 3, 27, 16, 22, 9, 30, 24, 8, 28, 15, 4, 21, 5, 10, 6, 14, 11]

For size 32, the time was 0.011588733000000004.

[29, 6, 15, 1, 27, 0, 2, 17, 22, 3, 10, 8, 31, 21, 30, 32, 25, 16, 24, 20, 7, 5, 26, 12, 4, 13, 28, 19, 9, 11, 23, 14, 18]

For size 33, the time was 0.015306015000000006.

[13, 6, 30, 27, 12, 31, 21, 11, 25, 28, 24, 20, 0, 29, 1, 10, 5, 23, 4, 26, 19, 3, 7, 15, 2, 22, 17, 8, 18, 32, 14, 33, 9, 16]

For size 34, the time was 0.026276612000000005.

[28, 10, 34, 27, 15, 13, 18, 0, 2, 12, 30, 32, 25, 14, 24, 11, 6, 31, 17, 1, 3, 26, 22, 8, 21, 4, 33, 29, 5, 20, 16, 19, 7, 9, 23]

For size 35, the time was 0.005784317999999983.

[5, 21, 26, 29, 32, 24, 3, 11, 9, 25, 35, 14, 12, 8, 6, 22, 34, 27, 20, 7, 19, 10, 1, 4, 33, 31, 16, 13, 2, 23, 28, 15, 17, 0, 30, 18]

For size 36, the time was 0.006401043999999995.

[17, 2, 32, 27, 24, 26, 14, 6, 3, 31, 23, 15, 4, 28, 21, 0, 30, 7, 1, 19, 36, 33, 25, 9, 5, 34, 10, 16, 22, 35, 12, 18, 20, 29, 11, 13, 8]

For size 37, the time was 0.030161737999999993.

[11, 32, 8, 2, 22, 3, 18, 34, 30, 33, 15, 37, 9, 23, 4, 31, 29, 17, 26, 10, 12, 36, 5, 24, 6, 27, 13, 16, 21, 25, 1, 19, 35, 7, 28, 0, 20, 14]

For size 38, the time was 0.00821913299999999.

[6, 24, 17, 0, 5, 29, 20, 11, 37, 31, 19, 16, 12, 9, 33, 22, 27, 15, 34, 1, 3, 7, 13, 36, 14, 35, 38, 26, 2, 21, 32, 8, 25, 28, 10, 23, 30, 4, 18]

For size 39, the time was 0.022381259999999986.

[31, 38, 22, 4, 16, 11, 15, 5, 34, 37, 18, 3, 25, 6, 39, 12, 10, 17, 0, 29, 24, 2, 33, 26, 8, 32, 21, 14, 7, 9, 13, 20, 28, 19, 36, 1, 35, 27, 23, 30]

For size 40, the time was 0.01958712900000001.

[39, 34, 0, 23, 25, 11, 1, 6, 36, 28, 4, 21, 8, 10, 17, 38, 29, 35, 18, 30, 5, 7, 2, 32, 40, 15, 31, 16, 20, 33, 37, 19, 9, 26, 13, 3, 27, 24, 22, 12, 14]

For size 41, the time was 0.11241900500000002.

[23, 38, 24, 13, 40, 26, 0, 6, 21, 33, 39, 9, 12, 1, 19, 35, 25, 4, 30, 3, 15, 7, 41, 34, 32, 27, 29, 10, 2, 22, 8, 16, 11, 37, 31, 5, 28, 17, 20, 14, 36, 18]

For size 42, the time was 0.010435326000000023.

[10, 1, 33, 21, 0, 13, 26, 12, 38, 35, 39, 6, 2, 14, 11, 7, 32, 20, 22, 40, 3, 34, 41, 8, 18, 37, 17, 25, 42, 31, 36, 30, 19, 5, 16, 23, 29, 4, 9, 28, 24, 27, 15]

For size 43, the time was 0.07198132300000004.

[20, 7, 0, 13, 28, 30, 1, 41, 5, 18, 15, 25, 14, 4, 42, 37, 39, 11, 3, 23, 19, 36, 22, 40, 10, 12, 33, 6, 21, 32, 38, 9, 43, 34, 16, 27, 2, 17, 8, 35, 24, 29, 31, 26]

For size 44, the time was 0.08529329200000008.

[1, 19, 6, 33, 11, 5, 0, 31, 39, 22, 42, 28, 41, 27, 9, 13, 32, 25, 37, 40, 43, 20, 2, 14, 10, 18, 3, 29, 7, 4, 15, 36, 38, 17, 30, 23, 26, 34, 8, 12, 21, 44, 24, 35, 16]

For size 45, the time was 0.01649186299999994.

[2, 25, 37, 35, 18, 8, 29, 1, 16, 22, 5, 39, 21, 3, 11, 36, 26, 28, 9, 17, 20, 7, 40, 0, 44, 32, 43, 31, 27, 41, 4, 6, 15, 34, 19, 13, 42, 24, 14, 10, 45, 33, 30, 23, 12, 38]

For size 46, the time was 0.013609577999999956.

[15, 17, 5, 34, 10, 43, 26, 37, 1, 27, 23, 16, 40, 22, 6, 4, 18, 42, 3, 41, 19, 29, 8, 30, 7, 44, 0, 14, 38, 33, 31, 45, 13, 24, 46, 32, 20, 35, 9, 39, 2, 36, 12, 21, 11, 25, 28]

For size 47, the time was 0.013204773999999975.

[33, 40, 3, 22, 16, 26, 21, 39, 5, 27, 34, 4, 46, 1, 42, 2, 45, 47, 20, 18, 28, 11, 17, 36, 41, 35, 8, 30, 7, 0, 37, 14, 23, 44, 9, 12, 32, 15, 43, 24, 10, 13, 31, 19, 25, 29, 38, 6]

For size 48, the time was 0.018249301999999967.

[30, 16, 47, 25, 12, 22, 6, 26, 45, 38, 8, 24, 2, 23, 46, 9, 27, 44, 36, 4, 43, 10, 48, 11, 21, 34, 0, 14, 40, 28, 37, 35, 33, 19, 3, 41, 20, 42, 31, 5, 32, 17, 13, 39, 7, 1, 18, 15, 29]

For size 49, the time was 0.09377935600000009.

[18, 43, 2, 9, 15, 34, 31, 10, 48, 32, 22, 46, 4, 1, 29, 49, 20, 3, 45, 28, 7, 37, 27, 17, 41, 23, 47, 11, 5, 39, 12, 38, 40, 13, 0, 25, 19, 8, 33, 36, 42, 26, 35, 16, 6, 21, 24, 14, 44, 30]

For size 50, the time was 0.017451683999999967.

[10, 50, 32, 4, 18, 28, 48, 2, 42, 30, 34, 36, 20, 33, 7, 14, 44, 3, 22, 26, 23, 43, 16, 13, 37, 40, 15, 25, 47, 8, 41, 27, 17, 49, 46, 0, 38, 11, 21, 31, 45, 1, 39, 19, 35, 12, 9, 29, 5, 24, 6]

For size 51, the time was 0.145551887.

[37, 17, 33, 45, 6, 22, 14, 48, 0, 7, 43, 30, 12, 27, 15, 50, 31, 2, 13, 23, 19, 1, 3, 44, 36, 38, 4, 9, 5, 32, 21, 51, 26, 40, 18, 10, 42, 47, 28, 25, 16, 8, 35, 11, 49, 41, 29, 34, 20, 46, 24, 39]

For size 52, the time was 0.017794427999999973.

[52, 5, 23, 36, 46, 49, 25, 48, 45, 2, 8, 29, 17, 44, 27, 32, 4, 18, 28, 9, 31, 43, 16, 7, 47, 51, 11, 0, 42, 37, 6, 34, 12, 30, 33, 50, 13, 19, 24, 39, 35, 1, 38, 26, 14, 3, 15, 21, 40, 10, 41, 22, 20]

For size 53, the time was 0.03144667199999995.

[5, 16, 29, 19, 17, 40, 26, 47, 50, 1, 33, 52, 7, 13, 32, 36, 12, 21, 11, 6, 0, 43, 25, 37, 53, 8, 45, 3, 34, 10, 39, 22, 44, 23, 31, 2, 46, 38, 4, 18, 28, 48, 41, 51, 30, 27, 35, 49, 42, 24, 20, 15, 9, 14]

For size 54, the time was 0.036496136999999984.

[43, 36, 48, 44, 10, 34, 25, 6, 50, 53, 2, 12, 52, 27, 21, 13, 41, 54, 4, 23, 46, 38, 32, 26, 0, 47, 49, 29, 40, 19, 8, 14, 45, 28, 7, 17, 51, 42, 31, 35, 15, 3, 11, 24, 16, 1, 5, 18, 22, 33, 30, 39, 37, 20, 9]

For size 55, the time was 0.040884093000000066.

[55, 23, 6, 26, 15, 17, 9, 37, 33, 53, 2, 27, 11, 7, 21, 54, 24, 35, 39, 52, 13, 49, 41, 36, 29, 14, 0, 4, 19, 43, 45, 3, 10, 16, 51, 30, 1, 8, 28, 40, 46, 20, 42, 31, 12, 47, 22, 44, 32, 18, 48, 38, 25, 34, 50, 5]

For size 56, the time was 0.028620452999999935.

[20, 44, 6, 33, 26, 17, 8, 56, 34, 9, 25, 12, 47, 40, 52, 43, 49, 24, 50, 29, 37, 3, 45, 15, 19, 30, 14, 0, 5, 10, 51, 18, 46, 39, 28, 11, 54, 7, 31, 55, 36, 13, 41, 32, 53, 42, 4, 38, 22, 2, 48, 1, 27, 21, 23, 16, 35]

For size 57, the time was 0.024638004000000047.

[38, 24, 32, 54, 51, 41, 22, 57, 23, 6, 13, 37, 49, 7, 33, 21, 55, 9, 25, 30, 15, 42, 47, 36, 17, 43, 34, 0, 5, 46, 28, 14, 20, 4, 44, 35, 50, 3, 27, 40, 45, 53, 11, 8, 12, 31, 16, 19, 2, 48, 52, 26, 56, 1, 39, 29, 18, 10]

For size 58, the time was 0.022011012999999968.

[23, 34, 32, 56, 0, 50, 40, 53, 14, 12, 15, 5, 19, 39, 29, 13, 48, 10, 30, 37, 9, 20, 57, 27, 49, 47, 55, 18, 11, 3, 6, 31, 46, 28, 35, 54, 1, 7, 2, 41, 25, 16, 51, 24, 52, 8, 38, 44, 58, 22, 4, 17, 42, 21, 33, 43, 36, 26, 45]

For size 59, the time was 0.03961397.

[18, 50, 52, 14, 2, 29, 25, 51, 31, 10, 47, 49, 17, 56, 27, 9, 28, 5, 45, 21, 42, 16, 37, 12, 32, 53, 23, 3, 59, 38, 46, 41, 36, 0, 55, 35, 11, 24, 8, 4, 57, 7, 13, 22, 40, 48, 6, 39, 54, 1, 33, 44, 30, 26, 34, 19, 15, 20, 43, 58]

For size 60, the time was 0.025530829000000033.

[3, 59, 26, 35, 43, 40, 6, 56, 5, 20, 32, 7, 53, 17, 30, 46, 52, 9, 16, 0, 49, 36, 13, 60, 54, 8, 10, 29, 22, 11, 44, 24, 21, 18, 41, 58, 48, 4, 33, 38, 12, 50, 47, 51, 23, 55, 2, 25, 34, 39, 14, 57, 28, 1, 42, 15, 31, 19, 45, 27, 37]

For size 61, the time was 0.20830559400000004.

[24, 44, 37, 34, 40, 21, 43, 57, 59, 38, 20, 50, 60, 14, 29, 7, 41, 61, 56, 17, 15, 39, 9, 45, 1, 4, 47, 13, 25, 12, 3, 48, 2, 33, 53, 28, 18, 51, 27, 52, 11, 35, 16, 19, 6, 30, 36, 22, 8, 55, 5, 54, 0, 49, 32, 46, 58, 23, 26, 31, 10, 42]

For size 62, the time was 0.030284095999999927.

[32, 55, 42, 48, 19, 1, 5, 20, 8, 12, 57, 18, 33, 56, 4, 7, 50, 29, 54, 24, 30, 39, 3, 40, 11, 16, 35, 41, 10, 49, 61, 0, 17, 26, 53, 51, 60, 25, 27, 13, 44, 36, 6, 14, 46, 31, 9, 45, 59, 22, 58, 52, 2, 47, 21, 34, 62, 37, 15, 43, 28, 38, 23]

For size 63, the time was 0.04301742200000014.

[9, 18, 2, 4, 46, 17, 51, 39, 28, 1, 45, 15, 35, 57, 62, 41, 56, 22, 42, 40, 13, 8, 30, 61, 27, 24, 48, 37, 53, 36, 63, 60, 0, 21, 10, 50, 5, 55, 20, 34, 59, 54, 3, 26, 19, 23, 16, 6, 38, 12, 29, 31, 58, 47, 52, 32, 11, 14, 25, 7, 44, 33, 43, 49]

For size 64, the time was 0.029232241999999964.

[32, 9, 18, 20, 10, 55, 28, 37, 61, 21, 49, 22, 33, 5, 11, 41, 62, 12, 46, 43, 60, 1, 63, 17, 44, 13, 25, 50, 15, 8, 19, 0, 23, 38, 47, 6, 54, 52, 36, 24, 42, 45, 4, 53, 58, 64, 30, 48, 2, 56, 16, 14, 35, 26, 3, 51, 59, 27, 40, 34, 7, 39, 29, 31, 57]

For size 65, the time was 0.08372379699999999.

[61, 29, 8, 59, 36, 14, 10, 49, 58, 34, 41, 28, 13, 50, 35, 23, 12, 16, 37, 64, 4, 26, 62, 18, 51, 60, 9, 5, 44, 42, 30, 3, 25, 55, 24, 65, 38, 31, 52, 7, 27, 56, 2, 46, 54, 33, 53, 6, 0, 20, 15, 63, 21, 17, 40, 32, 45, 48, 1, 22, 57, 43, 47, 19, 39, 11]

For size 66, the time was 0.04560456700000004.

[32, 47, 56, 15, 53, 39, 59, 55, 34, 20, 29, 11, 18, 64, 12, 28, 25, 42, 51, 26, 13, 4, 2, 41, 54, 61, 63, 23, 27, 6, 50, 45, 40, 62, 65, 58, 60, 1, 66, 7, 9, 44, 21, 38, 46, 30, 24, 37, 3, 43, 10, 5, 14, 57, 0, 36, 17, 31, 16, 8, 49, 33, 35, 19, 48, 22, 52]

For size 67, the time was 0.041837567999999825.

[19, 23, 58, 46, 48, 53, 1, 5, 37, 61, 64, 62, 59, 4, 11, 16, 40, 29, 18, 24, 8, 42, 12, 39, 52, 50, 6, 10, 2, 66, 56, 13, 9, 22, 45, 67, 30, 44, 0, 57, 63, 7, 17, 49, 28, 54, 31, 14, 41, 35, 15, 47, 55, 34, 26, 65, 60, 21, 27, 38, 33, 3, 32, 20, 51, 36, 25, 43]

For size 68, the time was 0.40295929699999977.

[46, 8, 62, 68, 58, 28, 21, 43, 59, 22, 0, 25, 7, 29, 34, 57, 2, 9, 42, 6, 37, 65, 67, 11, 35, 14, 55, 36, 38, 26, 31, 49, 64, 12, 56, 39, 18, 63, 3, 17, 61, 24, 33, 10, 40, 50, 48, 66, 54, 20, 53, 1, 16, 5, 23, 30, 41, 51, 15, 52, 32, 27, 60, 19, 45, 13, 4, 47, 44]

For size 69, the time was 0.03648943700000018.

[18, 61, 68, 35, 60, 40, 65, 10, 59, 64, 31, 47, 3, 21, 2, 4, 69, 27, 38, 34, 49, 7, 5, 37, 63, 25, 66, 50, 24, 19, 29, 9, 54, 46, 32, 16, 6, 44, 11, 0, 15, 33, 1, 14, 55, 30, 8, 56, 67, 51, 43, 45, 58, 48, 28, 13, 57, 23, 62, 39, 12, 17, 26, 20, 41, 52, 42, 22, 36, 53]

For size 70, the time was 0.04889271500000003.

[39, 44, 52, 41, 32, 70, 57, 25, 23, 63, 10, 12, 26, 21, 56, 9, 60, 49, 3, 38, 17, 31, 0, 30, 54, 24, 48, 61, 2, 35, 53, 68, 1, 29, 7, 55, 67, 13, 22, 4, 8, 43, 69, 34, 11, 16, 27, 51, 20, 66, 37, 40, 6, 65, 47, 14, 33, 36, 28, 45, 5, 59, 50, 18, 46, 15, 19, 42, 58, 64, 62]

For size 71, the time was 0.040131963000000326.

[45, 15, 33, 62, 10, 35, 9, 40, 23, 8, 28, 53, 63, 42, 34, 51, 41, 25, 2, 13, 39, 47, 68, 27, 71, 3, 66, 19, 56, 12, 30, 48, 55, 1, 22, 70, 26, 69, 29, 49, 21, 52, 44, 0, 60, 31, 5, 36, 6, 61, 57, 38, 18, 32, 24, 17, 65, 54, 43, 4, 20, 16, 58, 64, 7, 37, 59, 14, 50, 67, 46, 11]

For size 72, the time was 0.04007025499999983.

[38, 72, 67, 63, 30, 41, 69, 23, 54, 17, 19, 53, 13, 26, 64, 71, 40, 36, 15, 70, 52, 7, 61, 25, 34, 56, 9, 33, 48, 16, 1, 11, 50, 68, 27, 12, 35, 37, 32, 28, 65, 39, 2, 0, 55, 57, 5, 8, 62, 45, 42, 66, 43, 6, 58, 10, 59, 31, 21, 4, 44, 46, 29, 51, 20, 3, 47, 49, 22, 18, 60, 14, 24]

For size 73, the time was 0.15000155199999998.

[11, 30, 37, 52, 68, 45, 58, 21, 4, 56, 16, 64, 17, 71, 33, 23, 42, 35, 1, 14, 29, 19, 22, 69, 6, 28, 62, 15, 61, 54, 2, 63, 39, 67, 44, 34, 5, 8, 66, 51, 53, 18, 43, 70, 12, 65, 20, 40, 50, 24, 36, 31, 10, 26, 0, 25, 72, 13, 73, 38, 47, 55, 46, 48, 3, 41, 57, 59, 9, 7, 60, 49, 27, 32]

For size 74, the time was 0.09730269700000038.

[22, 42, 66, 60, 62, 8, 4, 1, 34, 53, 45, 50, 44, 40, 73, 58, 61, 12, 65, 72, 68, 18, 29, 56, 33, 9, 64, 38, 2, 7, 11, 0, 46, 26, 14, 47, 70, 74, 48, 30, 57, 5, 32, 51, 36, 63, 69, 20, 23, 15, 10, 21, 71, 52, 59, 37, 39, 24, 17, 55, 49, 67, 41, 35, 6, 28, 54, 43, 16, 31, 19, 25, 13, 3, 27]

For size 75, the time was 0.327770224.

[25, 72, 4, 53, 58, 2, 35, 40, 13, 56, 22, 63, 46, 14, 70, 23, 29, 55, 42, 9, 43, 3, 65, 32, 61, 17, 44, 21, 31, 50, 74, 12, 20, 18, 1, 62, 10, 51, 54, 39, 71, 45, 48, 7, 0, 60, 37, 19, 5, 26, 11, 68, 28, 24, 47, 75, 67, 52, 33, 66, 38, 15, 6, 8, 30, 16, 49, 36, 64, 27, 57, 69, 34, 41, 73, 59]

For size 76, the time was 0.09468346299999997.

[64, 44, 76, 11, 67, 10, 32, 35, 18, 59, 12, 20, 7, 68, 62, 2, 69, 58, 21, 34, 14, 53, 33, 23, 30, 47, 39, 1, 45, 71, 37, 70, 72, 54, 6, 51, 60, 75, 13, 66, 16, 61, 24, 73, 48, 38, 36, 46, 4, 0, 41, 29, 5, 17, 9, 22, 52, 3, 56, 25, 19, 8, 74, 31, 43, 57, 63, 28, 49, 42, 27, 55, 26, 50, 15, 40, 65]

For size 77, the time was 0.10955505499999996.

[5, 34, 26, 17, 3, 65, 4, 41, 69, 35, 28, 48, 54, 59, 1, 11, 46, 12, 50, 42, 36, 68, 10, 64, 27, 22, 8, 38, 18, 56, 75, 60, 23, 77, 73, 45, 0, 39, 19, 43, 76, 33, 61, 2, 51, 53, 66, 72, 15, 29, 63, 24, 21, 14, 25, 70, 62, 74, 6, 37, 49, 47, 7, 31, 40, 58, 67, 32, 20, 52, 44, 55, 57, 13, 16, 9, 30, 71]

For size 78, the time was 0.15904263300000032.

[11, 54, 47, 4, 20, 72, 42, 71, 55, 61, 52, 31, 41, 18, 6, 23, 28, 15, 56, 60, 19, 8, 66, 46, 78, 51, 39, 13, 53, 62, 26, 35, 64, 76, 74, 45, 21, 27, 29, 32, 3, 44, 17, 50, 63, 2, 5, 36, 12, 1, 77, 48, 69, 22, 75, 73, 33, 57, 37, 65, 43, 70, 24, 9, 58, 67, 7, 34, 0, 25, 30, 16, 38, 68, 10, 40, 14, 49, 59]

For size 79, the time was 0.2243524560000001.

[62, 38, 30, 9, 17, 48, 25, 53, 5, 57, 66, 61, 78, 23, 28, 26, 39, 75, 45, 63, 74, 13, 34, 41, 6, 21, 77, 51, 33, 22, 37, 40, 16, 4, 59, 3, 72, 7, 68, 1, 14, 42, 27, 31, 76, 35, 54, 12, 43, 64, 2, 73, 46, 36, 56, 71, 19, 0, 79, 18, 47, 50, 11, 32, 67, 65, 20, 58, 49, 15, 69, 44, 70, 24, 60, 10, 29, 52, 55, 8]

For size 80, the time was 0.06775697999999997.

[77, 18, 66, 71, 34, 20, 6, 35, 40, 23, 60, 46, 24, 15, 75, 5, 13, 68, 22, 14, 29, 74, 80, 16, 10, 67, 65, 38, 33, 69, 17, 4, 73, 12, 42, 64, 43, 50, 59, 76, 56, 0, 52, 21, 28, 37, 70, 9, 11, 1, 51, 2, 72, 27, 36, 3, 54, 63, 30, 48, 78, 32, 19, 8, 58, 61, 7, 44, 53, 49, 39, 62, 47, 31, 57, 25, 41, 26, 45, 55, 79]

For size 81, the time was 0.060611563999999785.

[30, 45, 78, 18, 24, 63, 9, 3, 8, 76, 23, 53, 20, 38, 46, 77, 1, 57, 4, 16, 67, 28, 50, 54, 34, 29, 19, 80, 6, 72, 56, 48, 12, 70, 13, 69, 0, 59, 25, 31, 79, 65, 40, 33, 73, 64, 11, 52, 62, 32, 61, 10, 75, 71, 66, 7, 58, 51, 47, 43, 15, 36, 5, 35, 2, 60, 17, 68, 26, 22, 14, 44, 81, 27, 21, 74, 37, 39, 55, 42, 49, 41]

For size 82, the time was 0.228585614.

[60, 28, 76, 7, 59, 34, 38, 17, 39, 79, 67, 50, 71, 55, 0, 69, 53, 5, 13, 47, 29, 24, 1, 35, 40, 25, 33, 21, 58, 4, 27, 45, 8, 77, 51, 3, 37, 6, 43, 23, 2, 75, 9, 68, 63, 56, 81, 20, 72, 57, 52, 36, 74, 18, 46, 19, 49, 80, 78, 31, 73, 48, 61, 54, 82, 26, 62, 12, 14, 11, 44, 22, 70, 16, 30, 65, 42, 66, 41, 15, 32, 64, 10]

For size 83, the time was 0.07345067500000013.

[11, 19, 4, 29, 37, 47, 23, 8, 43, 63, 18, 6, 62, 78, 76, 64, 61, 56, 1, 28, 10, 17, 82, 59, 15, 69, 33, 39, 81, 27, 12, 3, 80, 21, 55, 50, 52, 41, 70, 16, 71, 9, 20, 32, 72, 0, 83, 34, 51, 46, 30, 35, 13, 7, 68, 79, 5, 57, 25, 44, 53, 40, 75, 73, 38, 22, 65, 26, 2, 74, 14, 77, 24, 67, 36, 31, 45, 42, 49, 60, 66, 54, 48, 58]

For size 84, the time was 0.12448059600000017.

[46, 29, 53, 70, 61, 69, 75, 32, 2, 23, 12, 84, 35, 79, 0, 10, 38, 62, 48, 25, 51, 19, 65, 77, 33, 6, 27, 34, 76, 82, 47, 21, 9, 30, 72, 74, 55, 78, 11, 17, 73, 67, 54, 39, 57, 24, 56, 71, 68, 1, 18, 42, 37, 14, 43, 3, 20, 7, 40, 13, 81, 60, 80, 63, 26, 45, 59, 36, 31, 15, 44, 4, 64, 5, 49, 58, 52, 8, 16, 83, 50, 41, 66, 22, 28]

For size 85, the time was 0.08413776900000025.

[31, 35, 20, 57, 76, 21, 44, 70, 8, 2, 32, 75, 3, 59, 33, 24, 36, 50, 53, 15, 77, 9, 34, 51, 4, 66, 37, 52, 55, 61, 79, 81, 83, 54, 71, 5, 40, 11, 19, 82, 41, 85, 78, 42, 67, 69, 23, 18, 30, 43, 58, 56, 10, 45, 68, 84, 62, 1, 6, 74, 13, 64, 72, 47, 49, 17, 27, 65, 63, 26, 0, 73, 28, 80, 25, 29, 48, 12, 16, 38, 22, 7, 14, 46, 39, 60]

For size 86, the time was 0.09505864700000011.

[40, 16, 77, 51, 53, 8, 80, 63, 20, 14, 26, 78, 69, 47, 59, 85, 33, 45, 83, 57, 21, 76, 73, 52, 11, 86, 12, 71, 17, 24, 28, 62, 67, 32, 70, 48, 79, 7, 13, 27, 23, 68, 5, 65, 50, 10, 56, 3, 29, 31, 22, 41, 19, 60, 15, 9, 0, 82, 1, 44, 81, 75, 55, 42, 61, 84, 25, 18, 72, 49, 54, 39, 66, 46, 74, 37, 2, 30, 36, 43, 4, 6, 34, 38, 58, 35, 64]

For size 87, the time was 0.12449857899999994.

[45, 42, 3, 86, 68, 61, 20, 72, 23, 39, 54, 62, 70, 85, 0, 25, 65, 21, 41, 67, 83, 46, 35, 1, 71, 5, 29, 17, 37, 56, 47, 59, 82, 79, 34, 26, 13, 43, 49, 31, 76, 84, 12, 80, 63, 87, 64, 28, 2, 9, 58, 53, 4, 77, 52, 19, 22, 14, 11, 32, 55, 33, 38, 57, 69, 40, 10, 51, 16, 24, 66, 50, 30, 44, 81, 36, 7, 60, 75, 48, 8, 15, 27, 18, 73, 78, 6, 74]

For size 88, the time was 0.07534986599999982.

[8, 23, 15, 87, 58, 26, 3, 65, 42, 83, 59, 4, 40, 85, 49, 66, 25, 69, 88, 60, 80, 46, 86, 38, 29, 63, 13, 7, 12, 56, 5, 84, 34, 24, 30, 67, 0, 41, 6, 50, 64, 81, 52, 16, 10, 68, 75, 1, 36, 19, 57, 54, 71, 22, 33, 55, 17, 39, 53, 79, 74, 9, 47, 62, 27, 51, 11, 73, 14, 45, 82, 61, 70, 28, 2, 18, 48, 78, 72, 20, 37, 31, 21, 35, 43, 77, 44, 76, 32]

For size 89, the time was 0.5548646240000004.

[20, 76, 43, 22, 58, 70, 73, 44, 2, 31, 82, 84, 68, 46, 13, 59, 62, 12, 42, 80, 89, 55, 11, 5, 66, 41, 35, 87, 6, 52, 17, 1, 85, 51, 38, 7, 10, 67, 77, 64, 28, 16, 47, 83, 27, 88, 61, 54, 65, 78, 56, 49, 36, 32, 57, 8, 37, 48, 60, 14, 81, 71, 24, 4, 72, 26, 79, 53, 69, 25, 0, 63, 15, 50, 86, 21, 34, 45, 30, 3, 18, 40, 75, 19, 74, 39, 33, 29, 9, 23]

For size 90, the time was 0.2282654390000003.

[37, 47, 66, 23, 14, 11, 80, 88, 4, 62, 34, 51, 48, 40, 17, 27, 75, 68, 65, 61, 72, 69, 39, 24, 74, 56, 20, 90, 84, 12, 52, 89, 19, 42, 38, 79, 28, 82, 2, 60, 18, 13, 55, 50, 1, 70, 32, 22, 81, 0, 26, 77, 86, 21, 83, 46, 44, 10, 8, 73, 58, 16, 43, 25, 87, 5, 36, 78, 41, 54, 9, 71, 35, 33, 53, 64, 45, 76, 6, 31, 7, 30, 59, 63, 49, 67, 3, 29, 85, 15, 57]

For size 91, the time was 0.08847001200000015.

[36, 2, 80, 87, 4, 15, 68, 40, 6, 22, 42, 19, 34, 3, 28, 89, 69, 64, 46, 14, 83, 18, 62, 9, 0, 44, 1, 71, 77, 63, 47, 55, 70, 17, 26, 16, 32, 60, 20, 91, 82, 29, 21, 12, 31, 8, 66, 72, 13, 10, 49, 90, 35, 74, 57, 85, 27, 59, 65, 48, 78, 54, 88, 30, 75, 24, 7, 79, 11, 41, 25, 86, 76, 58, 51, 81, 50, 43, 23, 84, 37, 33, 73, 5, 52, 39, 56, 67, 61, 45, 38, 53]

For size 92, the time was 1.781932577.

[0, 14, 67, 47, 30, 40, 38, 68, 80, 16, 19, 50, 8, 61, 84, 79, 43, 15, 29, 34, 3, 22, 91, 1, 77, 17, 71, 41, 12, 78, 36, 74, 63, 37, 23, 75, 31, 66, 88, 21, 9, 46, 44, 28, 86, 33, 82, 57, 10, 2, 62, 69, 4, 20, 92, 85, 89, 7, 48, 56, 39, 55, 81, 27, 13, 24, 83, 5, 25, 60, 90, 87, 53, 49, 6, 52, 42, 76, 58, 72, 11, 51, 18, 54, 70, 26, 73, 32, 35, 45, 65, 59, 64]

For size 93, the time was 0.10794536199999971.

[8, 45, 66, 12, 57, 35, 33, 84, 21, 61, 3, 30, 44, 92, 83, 65, 79, 40, 60, 80, 78, 2, 53, 88, 13, 9, 77, 15, 23, 72, 76, 28, 31, 50, 62, 75, 7, 1, 34, 26, 41, 46, 24, 17, 10, 81, 91, 86, 38, 55, 85, 49, 67, 11, 39, 93, 82, 90, 36, 73, 29, 18, 74, 25, 20, 59, 52, 6, 14, 16, 70, 5, 51, 48, 0, 47, 68, 54, 37, 89, 71, 64, 43, 19, 87, 58, 4, 63, 32, 69, 27, 22, 42, 56]

For size 94, the time was 0.11102969600000012.

[13, 91, 22, 46, 62, 55, 93, 23, 1, 19, 94, 40, 20, 39, 88, 85, 71, 58, 80, 54, 74, 43, 60, 21, 31, 25, 75, 69, 81, 6, 42, 9, 29, 52, 24, 34, 82, 8, 86, 72, 79, 47, 14, 83, 76, 41, 64, 61, 73, 35, 78, 32, 10, 37, 3, 59, 18, 87, 45, 68, 17, 15, 50, 28, 4, 48, 51, 90, 11, 2, 65, 92, 66, 7, 33, 57, 5, 38, 0, 26, 53, 16, 49, 84, 89, 77, 36, 56, 63, 27, 70, 67, 44, 30, 12]

For size 95, the time was 0.3911381890000003.

[33, 52, 38, 17, 48, 16, 76, 91, 11, 18, 45, 49, 72, 95, 88, 74, 31, 69, 19, 26, 86, 40, 44, 68, 53, 46, 10, 80, 60, 94, 50, 56, 71, 83, 15, 84, 73, 28, 79, 29, 36, 13, 90, 70, 14, 55, 93, 1, 3, 32, 43, 59, 5, 42, 66, 23, 62, 81, 21, 35, 2, 12, 67, 6, 51, 39, 92, 25, 65, 82, 58, 75, 64, 89, 0, 22, 20, 34, 77, 4, 47, 9, 61, 54, 30, 41, 63, 37, 24, 87, 85, 8, 78, 57, 7, 27]

For size 96, the time was 0.16123625799999974.

[40, 94, 61, 59, 39, 48, 91, 1, 29, 62, 89, 41, 0, 22, 5, 14, 23, 84, 46, 92, 7, 66, 20, 65, 88, 11, 8, 79, 28, 3, 68, 60, 27, 17, 58, 13, 80, 53, 19, 57, 30, 74, 35, 82, 16, 49, 87, 36, 24, 96, 67, 15, 77, 54, 73, 34, 78, 69, 18, 90, 33, 12, 70, 83, 44, 4, 76, 63, 71, 75, 93, 86, 2, 6, 85, 43, 38, 9, 26, 42, 25, 21, 95, 37, 81, 50, 52, 56, 31, 47, 51, 32, 45, 10, 64, 72, 55]

For size 97, the time was 0.43932390799999954.

[42, 81, 55, 5, 10, 87, 13, 11, 91, 95, 31, 54, 48, 72, 86, 65, 8, 39, 78, 6, 67, 18, 0, 92, 25, 33, 40, 57, 61, 73, 7, 63, 84, 12, 44, 60, 19, 32, 50, 97, 89, 22, 93, 83, 35, 1, 30, 46, 77, 4, 69, 24, 21, 71, 88, 90, 76, 94, 75, 52, 14, 29, 51, 34, 3, 53, 17, 41, 38, 59, 96, 15, 68, 37, 79, 47, 74, 80, 43, 2, 66, 26, 82, 20, 36, 16, 62, 27, 70, 9, 49, 85, 45, 56, 23, 28, 58, 64]

For size 98, the time was 0.09268236600000002.

[15, 23, 69, 90, 70, 3, 76, 80, 47, 92, 78, 39, 71, 7, 51, 20, 6, 94, 29, 54, 1, 95, 10, 30, 81, 50, 25, 11, 14, 65, 72, 27, 45, 66, 44, 8, 18, 53, 91, 42, 74, 55, 2, 49, 22, 19, 67, 16, 12, 98, 9, 21, 93, 32, 97, 85, 83, 5, 75, 56, 64, 87, 33, 4, 59, 88, 31, 13, 86, 0, 37, 48, 84, 73, 61, 77, 34, 60, 63, 24, 41, 57, 62, 17, 28, 35, 79, 38, 40, 43, 46, 26, 58, 82, 36, 96, 68, 52, 89]

For size 99, the time was 1.1463343420000012.

[44, 57, 5, 73, 67, 26, 32, 1, 69, 80, 55, 25, 95, 90, 96, 3, 35, 71, 14, 92, 15, 87, 24, 46, 72, 38, 53, 28, 11, 21, 62, 59, 2, 98, 6, 77, 20, 16, 48, 86, 64, 39, 58, 18, 41, 22, 68, 7, 0, 23, 9, 50, 42, 83, 97, 19, 85, 30, 63, 94, 8, 12, 40, 54, 27, 65, 4, 76, 99, 49, 13, 75, 89, 79, 60, 51, 84, 88, 31, 91, 36, 74, 37, 45, 66, 52, 93, 10, 17, 43, 34, 56, 81, 61, 29, 82, 33, 78, 47, 70]

For size 100, the time was 0.3220973330000003.

[69, 49, 51, 24, 8, 55, 78, 33, 38, 15, 44, 66, 21, 84, 3, 99, 40, 94, 97, 13, 98, 83, 0, 43, 56, 18, 91, 65, 2, 54, 28, 30, 71, 92, 41, 75, 63, 14, 81, 26, 19, 86, 37, 4, 27, 36, 61, 23, 90, 85, 46, 11, 89, 10, 35, 67, 74, 80, 77, 39, 88, 62, 31, 79, 64, 96, 1, 20, 17, 9, 87, 29, 82, 76, 16, 72, 60, 25, 68, 45, 52, 5, 70, 48, 95, 93, 100, 58, 7, 32, 53, 73, 59, 47, 22, 34, 12, 42, 57, 6, 50]

For size 101, the time was 0.10301120199999936.

[84, 50, 40, 19, 34, 52, 92, 67, 96, 81, 86, 14, 54, 66, 79, 85, 64, 41, 53, 9, 65, 31, 91, 10, 16, 82, 94, 70, 99, 73, 32, 36, 33, 37, 3, 57, 17, 68, 5, 71, 98, 0, 8, 21, 80, 44, 100, 22, 43, 62, 38, 78, 24, 72, 61, 101, 95, 30, 23, 13, 74, 47, 6, 15, 55, 49, 46, 63, 77, 48, 7, 12, 1, 93, 97, 75, 88, 59, 89, 76, 29, 51, 90, 45, 11, 18, 20, 35, 28, 60, 4, 27, 69, 87, 58, 26, 39, 42, 56, 2, 83, 25]

For size 102, the time was 0.122655807000001.

[51, 13, 26, 12, 83, 79, 3, 101, 93, 54, 14, 63, 69, 5, 44, 56, 60, 50, 78, 27, 87, 59, 23, 88, 53, 96, 47, 61, 97, 43, 55, 58, 11, 86, 57, 33, 84, 32, 9, 2, 46, 99, 18, 7, 80, 94, 36, 19, 74, 77, 65, 42, 98, 49, 0, 92, 6, 73, 76, 24, 30, 100, 82, 95, 71, 68, 66, 102, 41, 10, 29, 81, 40, 22, 1, 17, 89, 28, 67, 37, 62, 48, 15, 45, 72, 21, 91, 16, 25, 34, 4, 52, 39, 70, 20, 64, 31, 90, 85, 38, 75, 35, 8]

For size 103, the time was 0.12419533000000094.

[75, 3, 16, 4, 36, 10, 84, 91, 64, 20, 69, 85, 49, 25, 2, 31, 40, 93, 89, 54, 101, 78, 80, 42, 68, 22, 11, 21, 95, 65, 39, 53, 35, 72, 88, 83, 99, 58, 45, 94, 1, 17, 38, 63, 44, 26, 86, 100, 30, 92, 9, 46, 90, 32, 14, 27, 71, 19, 76, 50, 0, 47, 8, 41, 98, 12, 70, 57, 96, 5, 59, 79, 43, 103, 97, 28, 13, 60, 48, 37, 6, 74, 66, 82, 61, 102, 7, 52, 56, 24, 29, 23, 33, 73, 81, 87, 34, 15, 55, 62, 51, 67, 18, 77]

For size 104, the time was 0.13554005299999972.

[48, 26, 40, 85, 76, 54, 58, 31, 8, 95, 11, 65, 67, 77, 3, 21, 62, 101, 55, 52, 47, 87, 33, 84, 44, 68, 73, 12, 38, 82, 13, 19, 74, 20, 96, 6, 27, 102, 14, 23, 103, 80, 15, 17, 88, 42, 51, 7, 64, 61, 63, 69, 9, 45, 18, 30, 49, 34, 89, 66, 10, 39, 60, 53, 32, 79, 36, 75, 91, 0, 104, 24, 5, 28, 43, 97, 93, 72, 16, 78, 29, 83, 4, 92, 35, 100, 90, 50, 46, 56, 70, 1, 71, 22, 37, 57, 99, 41, 94, 81, 86, 2, 25, 59, 98]

For size 105, the time was 0.1671683220000002.

[59, 35, 5, 103, 60, 56, 17, 96, 85, 67, 52, 101, 92, 77, 66, 30, 51, 29, 7, 12, 53, 71, 38, 61, 78, 49, 104, 95, 0, 10, 75, 44, 69, 81, 48, 37, 34, 42, 6, 79, 23, 25, 36, 74, 41, 91, 2, 11, 65, 39, 94, 72, 105, 62, 3, 13, 43, 83, 19, 24, 40, 47, 54, 70, 86, 97, 89, 4, 98, 57, 16, 18, 80, 102, 84, 45, 20, 32, 73, 50, 58, 87, 64, 15, 88, 9, 1, 8, 31, 22, 90, 82, 55, 68, 93, 46, 14, 63, 33, 100, 28, 26, 76, 99, 21, 27]

For size 106, the time was 0.12060322800000023.

[68, 77, 5, 73, 66, 21, 75, 30, 50, 40, 8, 22, 13, 106, 97, 59, 83, 103, 69, 10, 24, 18, 85, 71, 45, 70, 41, 57, 28, 63, 29, 105, 78, 15, 26, 47, 93, 84, 62, 91, 7, 31, 19, 37, 102, 88, 17, 3, 9, 99, 64, 2, 89, 94, 80, 35, 81, 92, 60, 38, 65, 79, 82, 25, 86, 43, 11, 56, 49, 44, 34, 55, 100, 42, 101, 0, 46, 32, 87, 74, 6, 98, 95, 20, 12, 72, 96, 14, 36, 4, 1, 76, 51, 23, 48, 67, 16, 104, 58, 33, 39, 53, 90, 61, 27, 54, 52]

For size 107, the time was 0.14069812400000004.

[86, 16, 97, 50, 69, 1, 75, 61, 24, 45, 42, 94, 84, 95, 32, 89, 62, 14, 7, 5, 82, 100, 57, 104, 85, 67, 64, 58, 21, 34, 59, 102, 90, 39, 83, 23, 48, 40, 19, 22, 88, 107, 13, 93, 43, 15, 66, 54, 76, 31, 63, 91, 79, 47, 44, 4, 35, 10, 30, 51, 2, 36, 87, 6, 105, 49, 17, 28, 3, 106, 68, 11, 73, 92, 29, 60, 80, 103, 65, 46, 27, 81, 99, 74, 98, 33, 25, 53, 38, 52, 101, 56, 37, 0, 70, 72, 20, 77, 8, 55, 78, 96, 71, 41, 26, 18, 9, 12]

For size 108, the time was 0.6093850930000002.

[14, 49, 84, 61, 76, 57, 40, 33, 25, 93, 21, 17, 69, 44, 10, 60, 79, 73, 23, 102, 63, 6, 66, 74, 11, 71, 99, 65, 26, 48, 92, 39, 72, 16, 81, 28, 38, 104, 97, 94, 32, 105, 64, 0, 24, 13, 53, 51, 101, 52, 77, 5, 1, 18, 31, 36, 55, 82, 58, 19, 80, 100, 103, 45, 2, 88, 27, 95, 37, 4, 107, 87, 42, 86, 62, 85, 108, 78, 9, 3, 54, 29, 35, 98, 96, 43, 70, 7, 106, 75, 22, 41, 59, 30, 89, 34, 47, 91, 50, 8, 90, 56, 68, 15, 20, 67, 46, 12, 83]

For size 109, the time was 0.12853744599999928.

[18, 101, 63, 85, 21, 43, 35, 100, 74, 15, 25, 20, 32, 97, 71, 69, 81, 60, 28, 1, 16, 29, 84, 70, 105, 38, 82, 48, 76, 24, 98, 67, 64, 83, 65, 108, 19, 5, 13, 8, 52, 80, 58, 6, 104, 56, 99, 39, 26, 73, 23, 109, 0, 102, 3, 54, 57, 92, 77, 93, 11, 106, 4, 68, 61, 53, 96, 94, 66, 14, 44, 10, 22, 17, 59, 40, 36, 34, 9, 79, 46, 103, 37, 90, 7, 62, 88, 78, 50, 41, 27, 12, 45, 107, 75, 49, 86, 30, 91, 55, 72, 31, 89, 2, 47, 51, 33, 87, 42, 95]

For size 110, the time was 0.13006921200000043.

[12, 59, 2, 26, 48, 79, 82, 76, 3, 99, 60, 33, 54, 93, 53, 109, 61, 104, 23, 81, 19, 50, 41, 102, 56, 90, 77, 9, 22, 110, 87, 24, 65, 25, 58, 95, 21, 16, 73, 29, 94, 8, 20, 30, 35, 46, 67, 43, 64, 38, 57, 103, 49, 28, 11, 17, 86, 91, 105, 97, 100, 13, 89, 31, 45, 101, 27, 70, 42, 71, 98, 96, 78, 84, 4, 51, 0, 85, 18, 10, 6, 14, 62, 92, 88, 68, 72, 32, 108, 40, 74, 106, 7, 5, 44, 36, 69, 107, 37, 47, 55, 1, 34, 66, 39, 15, 83, 63, 80, 52, 75]

For size 111, the time was 0.14782767200000002.

[60, 54, 28, 13, 31, 73, 39, 55, 97, 38, 81, 95, 34, 80, 1, 85, 5, 51, 105, 76, 108, 63, 2, 101, 33, 90, 8, 21, 102, 68, 107, 10, 24, 19, 9, 40, 111, 65, 84, 37, 109, 26, 103, 66, 0, 18, 23, 12, 100, 32, 70, 20, 36, 89, 50, 74, 106, 15, 59, 104, 98, 79, 17, 94, 72, 86, 44, 92, 6, 43, 3, 47, 35, 16, 22, 77, 7, 49, 78, 96, 87, 93, 53, 42, 99, 14, 30, 27, 25, 88, 71, 82, 52, 46, 61, 83, 57, 4, 48, 69, 11, 91, 41, 29, 110, 67, 58, 64, 62, 45, 56, 75]

For size 112, the time was 1.4075196830000003.

[14, 58, 86, 99, 40, 106, 18, 72, 28, 39, 74, 103, 17, 73, 75, 53, 47, 108, 101, 41, 12, 70, 23, 71, 61, 44, 111, 6, 49, 32, 110, 31, 109, 102, 79, 37, 35, 21, 94, 66, 26, 93, 9, 84, 59, 4, 1, 5, 81, 34, 89, 80, 24, 46, 3, 52, 100, 33, 38, 87, 7, 107, 2, 15, 51, 105, 92, 30, 91, 78, 36, 22, 83, 55, 64, 48, 54, 112, 95, 13, 68, 20, 63, 96, 82, 27, 77, 11, 104, 97, 43, 62, 25, 57, 88, 56, 85, 29, 10, 76, 69, 42, 19, 98, 60, 67, 16, 90, 8, 45, 0, 65, 50]

For size 113, the time was 0.14115723000000102.

[91, 111, 54, 90, 71, 33, 65, 1, 97, 78, 45, 15, 106, 87, 37, 29, 2, 93, 99, 105, 17, 41, 60, 44, 85, 76, 5, 68, 100, 61, 59, 94, 20, 89, 96, 48, 18, 67, 77, 107, 8, 28, 69, 53, 35, 92, 22, 14, 9, 58, 52, 26, 102, 31, 4, 109, 80, 112, 50, 19, 16, 73, 79, 11, 62, 64, 7, 13, 110, 70, 88, 23, 98, 12, 108, 86, 83, 72, 21, 42, 113, 66, 24, 36, 103, 3, 6, 46, 39, 27, 30, 57, 75, 47, 63, 101, 104, 34, 25, 56, 81, 74, 51, 0, 10, 38, 32, 84, 55, 43, 40, 95, 82, 49]

For size 114, the time was 0.1453096390000006.

[4, 65, 35, 58, 5, 110, 77, 9, 56, 34, 112, 16, 94, 10, 36, 0, 23, 63, 27, 40, 18, 111, 109, 29, 51, 62, 91, 85, 8, 20, 107, 46, 26, 113, 70, 92, 24, 60, 2, 81, 83, 75, 28, 93, 114, 64, 59, 6, 99, 49, 32, 98, 69, 102, 80, 48, 30, 17, 50, 104, 12, 15, 57, 101, 43, 97, 105, 25, 71, 100, 84, 39, 88, 21, 37, 3, 72, 22, 90, 14, 61, 89, 53, 13, 1, 55, 106, 38, 41, 67, 11, 74, 103, 68, 19, 79, 86, 54, 108, 31, 73, 66, 42, 47, 76, 82, 95, 7, 45, 96, 33, 87, 78, 44, 52]

For size 115, the time was 0.2520409430000008.

[75, 85, 24, 69, 108, 54, 11, 67, 22, 71, 82, 36, 3, 103, 31, 25, 50, 73, 15, 6, 51, 66, 74, 39, 19, 80, 102, 9, 81, 13, 113, 109, 41, 44, 93, 99, 86, 56, 10, 28, 105, 29, 111, 33, 88, 65, 5, 68, 76, 106, 57, 91, 2, 45, 47, 96, 12, 4, 94, 107, 0, 104, 100, 48, 97, 23, 101, 32, 1, 37, 21, 77, 53, 115, 8, 55, 78, 7, 72, 92, 112, 110, 20, 98, 16, 38, 17, 87, 84, 58, 114, 95, 49, 42, 14, 34, 60, 52, 35, 61, 46, 62, 18, 79, 40, 59, 83, 90, 30, 27, 70, 89, 64, 26, 43, 63]

For size 116, the time was 1.7476138460000001.

[96, 58, 85, 28, 91, 1, 43, 83, 47, 64, 61, 71, 2, 17, 79, 77, 9, 29, 98, 60, 8, 106, 111, 92, 114, 49, 68, 27, 20, 36, 51, 5, 75, 12, 109, 40, 88, 13, 65, 23, 108, 59, 112, 78, 102, 46, 10, 87, 35, 93, 7, 107, 66, 33, 86, 81, 54, 72, 41, 11, 16, 99, 44, 69, 97, 84, 14, 90, 104, 50, 80, 37, 103, 62, 94, 42, 89, 45, 63, 19, 21, 39, 116, 30, 53, 82, 3, 15, 0, 25, 55, 18, 67, 4, 31, 57, 95, 113, 76, 32, 26, 52, 105, 74, 24, 6, 115, 101, 22, 100, 34, 70, 73, 38, 56, 110, 48]

For size 117, the time was 0.15424669900000154.

[78, 2, 24, 80, 62, 49, 41, 94, 64, 88, 5, 76, 8, 20, 48, 28, 13, 31, 61, 103, 90, 25, 73, 112, 92, 106, 54, 82, 70, 39, 10, 34, 53, 60, 21, 84, 108, 68, 36, 38, 114, 117, 44, 116, 27, 30, 69, 100, 93, 65, 107, 105, 0, 37, 4, 63, 56, 72, 50, 40, 29, 33, 99, 6, 3, 71, 83, 57, 14, 35, 26, 42, 1, 98, 18, 113, 115, 66, 97, 9, 47, 19, 46, 23, 7, 109, 77, 15, 74, 101, 58, 43, 45, 111, 51, 22, 89, 55, 52, 104, 11, 110, 79, 81, 16, 87, 67, 95, 102, 12, 59, 85, 75, 86, 32, 17, 91, 96]

For size 118, the time was 0.33138489399999926.

[28, 24, 81, 112, 65, 4, 97, 17, 109, 29, 12, 77, 70, 27, 54, 11, 21, 111, 62, 36, 38, 22, 25, 9, 63, 37, 52, 98, 85, 19, 61, 78, 110, 74, 96, 60, 91, 114, 1, 14, 16, 71, 42, 80, 90, 49, 18, 115, 56, 103, 66, 15, 105, 20, 102, 2, 7, 106, 108, 13, 26, 53, 44, 58, 35, 0, 73, 94, 100, 57, 31, 69, 117, 92, 95, 84, 55, 45, 3, 6, 30, 64, 104, 116, 113, 50, 99, 33, 79, 48, 59, 5, 41, 87, 47, 23, 107, 75, 34, 51, 8, 10, 86, 88, 101, 39, 76, 46, 82, 89, 83, 67, 118, 68, 32, 72, 93, 43, 40]

For size 119, the time was 0.16418800900000008.

[38, 33, 9, 47, 82, 26, 58, 11, 80, 14, 59, 21, 99, 7, 105, 108, 111, 12, 119, 36, 19, 76, 86, 89, 109, 60, 18, 78, 48, 102, 3, 16, 62, 116, 87, 103, 13, 117, 49, 63, 5, 39, 6, 91, 27, 110, 68, 75, 4, 40, 65, 93, 113, 43, 2, 55, 90, 25, 81, 84, 41, 23, 8, 72, 34, 0, 17, 46, 22, 106, 88, 97, 115, 31, 52, 50, 107, 73, 114, 51, 1, 95, 70, 96, 77, 104, 92, 37, 15, 118, 98, 94, 61, 67, 54, 79, 20, 83, 30, 101, 66, 44, 69, 74, 32, 64, 35, 56, 10, 85, 71, 29, 45, 100, 53, 112, 57, 42, 24, 28]

For size 120, the time was 0.21377044099999765.

[13, 89, 21, 113, 16, 34, 120, 46, 99, 66, 64, 37, 9, 107, 35, 52, 86, 70, 51, 98, 118, 97, 11, 69, 42, 117, 8, 74, 30, 22, 92, 26, 105, 94, 62, 95, 10, 3, 25, 50, 72, 44, 67, 0, 109, 36, 77, 85, 115, 90, 93, 60, 32, 20, 1, 4, 111, 84, 78, 55, 110, 83, 43, 17, 100, 33, 6, 101, 2, 59, 119, 12, 114, 73, 47, 82, 48, 14, 41, 54, 45, 27, 80, 68, 15, 71, 102, 104, 18, 65, 5, 96, 91, 7, 53, 79, 88, 61, 112, 49, 106, 39, 57, 63, 75, 56, 28, 24, 116, 87, 19, 81, 23, 58, 108, 103, 29, 38, 76, 31, 40]

For size 121, the time was 0.19527501199999975.

[19, 119, 61, 84, 98, 39, 66, 74, 85, 110, 65, 103, 62, 60, 75, 83, 78, 88, 111, 117, 48, 22, 121, 69, 82, 105, 58, 101, 17, 11, 0, 10, 47, 13, 56, 100, 41, 1, 107, 25, 30, 77, 115, 54, 96, 43, 6, 14, 112, 2, 67, 64, 3, 80, 31, 104, 52, 108, 93, 37, 9, 4, 50, 71, 57, 81, 106, 91, 33, 92, 55, 7, 97, 34, 12, 79, 23, 46, 120, 16, 20, 99, 27, 8, 90, 18, 89, 116, 102, 21, 63, 53, 86, 113, 44, 76, 35, 51, 26, 40, 95, 49, 24, 38, 87, 114, 5, 59, 118, 109, 73, 68, 70, 72, 28, 32, 36, 15, 94, 45, 29, 42]

For size 122, the time was 0.20012172299999875.

[47, 61, 70, 18, 30, 104, 14, 122, 10, 57, 59, 66, 42, 117, 93, 107, 1, 26, 8, 114, 16, 9, 36, 63, 81, 89, 99, 40, 11, 49, 5, 96, 102, 50, 53, 116, 60, 103, 37, 67, 12, 22, 2, 74, 111, 108, 13, 48, 55, 120, 106, 72, 121, 88, 58, 105, 4, 109, 24, 52, 76, 3, 27, 95, 100, 19, 84, 6, 97, 28, 46, 17, 43, 110, 101, 45, 118, 68, 90, 112, 64, 54, 77, 94, 20, 25, 7, 21, 33, 39, 0, 71, 115, 31, 119, 51, 82, 75, 92, 62, 87, 38, 91, 29, 34, 32, 41, 35, 23, 83, 78, 15, 73, 44, 65, 56, 113, 86, 80, 98, 69, 85, 79]

For size 123, the time was 0.18129836099999963.

[73, 78, 85, 100, 27, 40, 82, 29, 10, 110, 17, 96, 106, 46, 117, 84, 22, 74, 79, 75, 122, 65, 38, 87, 90, 49, 94, 108, 7, 19, 25, 102, 58, 80, 28, 69, 72, 21, 42, 2, 52, 95, 104, 101, 81, 60, 31, 76, 68, 112, 48, 3, 18, 15, 67, 56, 53, 99, 107, 89, 115, 14, 39, 77, 109, 57, 12, 118, 121, 26, 120, 1, 83, 98, 41, 92, 9, 6, 105, 55, 88, 51, 113, 34, 0, 54, 86, 62, 5, 11, 63, 33, 20, 103, 44, 50, 70, 32, 4, 64, 47, 61, 111, 24, 123, 59, 37, 71, 97, 45, 66, 30, 16, 116, 23, 35, 43, 13, 8, 91, 119, 36, 93, 114]

For size 124, the time was 0.20899178599999857.

[115, 67, 107, 122, 33, 82, 116, 59, 64, 105, 23, 72, 61, 44, 28, 83, 48, 15, 38, 35, 20, 113, 101, 120, 75, 30, 5, 66, 50, 119, 17, 96, 92, 1, 68, 45, 27, 2, 56, 21, 110, 87, 90, 54, 103, 14, 93, 121, 88, 70, 40, 26, 22, 8, 118, 124, 97, 13, 18, 95, 114, 4, 89, 106, 36, 41, 111, 109, 52, 0, 16, 37, 91, 62, 57, 81, 3, 63, 6, 76, 117, 31, 108, 12, 112, 79, 98, 9, 49, 29, 11, 42, 34, 74, 65, 43, 7, 55, 94, 100, 104, 10, 69, 39, 84, 78, 60, 71, 53, 86, 102, 19, 32, 47, 73, 123, 46, 24, 80, 51, 25, 99, 85, 58, 77]

For size 125, the time was 0.1894854059999993.

[73, 11, 58, 107, 87, 57, 49, 23, 86, 0, 8, 121, 111, 108, 54, 30, 123, 91, 2, 32, 20, 81, 48, 120, 94, 64, 46, 1, 47, 71, 68, 97, 31, 5, 102, 69, 122, 27, 59, 116, 9, 24, 106, 72, 14, 67, 4, 114, 29, 125, 56, 52, 113, 85, 39, 35, 3, 44, 89, 36, 21, 63, 110, 100, 88, 101, 19, 62, 118, 104, 41, 28, 15, 10, 79, 82, 51, 74, 124, 109, 34, 45, 55, 76, 50, 7, 78, 33, 43, 98, 93, 103, 117, 75, 112, 40, 13, 53, 115, 18, 96, 90, 17, 38, 66, 37, 84, 6, 119, 16, 70, 105, 80, 99, 42, 25, 65, 61, 77, 95, 83, 12, 60, 26, 22, 92]

For size 126, the time was 0.2850525260000012.

[6, 48, 119, 126, 28, 5, 80, 37, 9, 82, 117, 106, 57, 7, 16, 53, 107, 24, 59, 120, 68, 43, 124, 62, 73, 29, 89, 0, 15, 55, 81, 41, 103, 45, 98, 3, 22, 87, 118, 122, 67, 77, 84, 19, 36, 121, 21, 66, 56, 116, 104, 32, 96, 114, 123, 2, 52, 74, 38, 14, 78, 92, 27, 88, 12, 79, 8, 39, 11, 26, 105, 69, 109, 25, 90, 44, 108, 33, 42, 112, 10, 60, 91, 72, 46, 17, 71, 102, 13, 47, 4, 58, 30, 76, 20, 65, 125, 85, 64, 110, 113, 51, 1, 93, 101, 83, 97, 35, 111, 70, 23, 34, 94, 50, 54, 99, 18, 61, 40, 31, 115, 95, 75, 100, 63, 49, 86]

For size 127, the time was 0.20481716100000114.

[54, 90, 5, 59, 74, 25, 121, 92, 87, 44, 37, 119, 20, 99, 112, 79, 18, 11, 32, 115, 95, 126, 61, 66, 56, 63, 67, 49, 23, 17, 42, 2, 108, 24, 26, 35, 84, 88, 103, 57, 64, 69, 16, 62, 89, 97, 13, 118, 38, 80, 110, 50, 120, 100, 1, 51, 28, 7, 39, 15, 71, 58, 117, 107, 9, 72, 0, 83, 55, 105, 75, 40, 36, 122, 47, 4, 109, 81, 76, 125, 29, 21, 124, 45, 19, 114, 123, 3, 73, 98, 10, 52, 6, 53, 111, 96, 82, 127, 113, 77, 22, 12, 33, 78, 14, 31, 65, 48, 41, 93, 86, 104, 91, 70, 68, 85, 8, 60, 46, 43, 102, 27, 30, 106, 101, 34, 94, 116]

For size 128, the time was 0.20377800099999988.

[125, 31, 97, 52, 128, 117, 72, 98, 121, 21, 29, 40, 92, 68, 56, 37, 44, 93, 1, 107, 13, 118, 12, 122, 11, 34, 112, 26, 115, 8, 55, 116, 101, 84, 108, 76, 46, 126, 42, 91, 100, 33, 58, 51, 83, 78, 63, 2, 105, 16, 64, 57, 96, 124, 43, 18, 79, 0, 9, 61, 19, 27, 53, 38, 110, 99, 62, 114, 103, 82, 20, 4, 49, 127, 28, 90, 70, 109, 6, 106, 7, 69, 67, 35, 23, 85, 104, 88, 74, 94, 47, 111, 14, 73, 120, 3, 65, 59, 54, 25, 60, 15, 80, 41, 75, 102, 87, 24, 119, 50, 86, 81, 77, 71, 5, 39, 89, 17, 113, 22, 123, 36, 66, 32, 45, 48, 10, 95, 30]

For size 129, the time was 0.24914949000000064.

[41, 109, 97, 91, 116, 51, 63, 27, 126, 49, 104, 86, 67, 11, 118, 75, 20, 94, 21, 77, 81, 55, 120, 32, 129, 24, 115, 18, 84, 92, 23, 37, 70, 82, 96, 17, 12, 128, 43, 54, 52, 72, 9, 1, 13, 125, 112, 0, 25, 56, 102, 68, 88, 123, 6, 19, 48, 29, 85, 87, 40, 22, 76, 98, 107, 124, 114, 99, 69, 90, 121, 58, 50, 106, 119, 3, 73, 79, 66, 60, 36, 44, 93, 122, 33, 127, 108, 4, 35, 89, 38, 110, 105, 111, 78, 14, 7, 65, 83, 8, 95, 5, 16, 47, 59, 26, 2, 117, 28, 45, 100, 46, 39, 42, 74, 53, 31, 103, 30, 113, 15, 62, 64, 10, 61, 71, 34, 57, 101, 80]

For size 130, the time was 0.23583011499999884.

[59, 46, 32, 53, 16, 22, 31, 72, 75, 48, 23, 105, 13, 88, 38, 113, 115, 80, 33, 12, 40, 127, 118, 85, 90, 71, 6, 112, 116, 43, 63, 39, 124, 54, 98, 126, 4, 84, 8, 70, 66, 109, 21, 83, 125, 44, 95, 11, 7, 100, 17, 34, 28, 0, 110, 20, 18, 130, 119, 45, 58, 50, 68, 97, 106, 42, 37, 96, 9, 122, 25, 103, 30, 47, 64, 60, 10, 15, 94, 123, 49, 108, 93, 79, 102, 3, 26, 81, 107, 92, 27, 51, 73, 129, 19, 52, 87, 99, 82, 35, 104, 111, 77, 56, 1, 128, 67, 114, 14, 24, 61, 65, 55, 41, 86, 120, 89, 117, 74, 36, 62, 2, 5, 101, 29, 69, 121, 57, 91, 78, 76]

For size 131, the time was 0.2634755829999982.

[42, 75, 95, 86, 74, 107, 81, 115, 33, 28, 40, 118, 68, 128, 6, 56, 97, 67, 3, 66, 37, 7, 55, 10, 69, 2, 60, 131, 44, 82, 112, 108, 14, 71, 100, 102, 62, 124, 52, 91, 23, 19, 78, 51, 8, 16, 13, 48, 120, 76, 65, 122, 21, 101, 103, 20, 79, 50, 104, 117, 59, 27, 123, 5, 54, 119, 70, 110, 15, 57, 125, 39, 92, 18, 47, 99, 26, 73, 129, 35, 89, 121, 61, 88, 105, 38, 96, 94, 11, 111, 90, 9, 83, 34, 46, 113, 98, 36, 127, 4, 32, 77, 53, 114, 116, 43, 64, 1, 41, 29, 80, 17, 87, 85, 31, 72, 30, 0, 49, 12, 45, 58, 22, 24, 130, 84, 106, 25, 109, 63, 93, 126]

For size 132, the time was 0.2205202220000011.

[66, 109, 26, 76, 110, 129, 56, 15, 111, 29, 96, 43, 37, 3, 132, 77, 127, 40, 9, 71, 35, 4, 11, 23, 0, 112, 130, 32, 89, 104, 34, 94, 128, 44, 5, 68, 117, 91, 50, 74, 114, 67, 51, 120, 28, 6, 124, 22, 10, 95, 46, 115, 8, 118, 72, 42, 49, 98, 101, 53, 70, 75, 107, 21, 63, 116, 85, 65, 14, 17, 125, 2, 19, 1, 48, 82, 103, 90, 126, 59, 119, 87, 47, 12, 57, 102, 88, 36, 54, 105, 30, 7, 121, 16, 122, 20, 81, 24, 41, 25, 131, 79, 99, 106, 61, 93, 58, 45, 80, 31, 73, 97, 113, 92, 78, 33, 84, 18, 100, 39, 52, 55, 27, 38, 83, 64, 86, 69, 123, 62, 60, 108, 13]

For size 133, the time was 0.3780836289999989.

[22, 115, 75, 62, 47, 35, 39, 67, 10, 80, 5, 99, 60, 4, 120, 20, 92, 113, 126, 44, 123, 119, 85, 98, 23, 65, 8, 43, 108, 53, 51, 82, 87, 69, 88, 117, 102, 29, 17, 56, 93, 127, 73, 110, 40, 59, 131, 112, 38, 13, 97, 25, 1, 76, 34, 14, 133, 107, 9, 26, 64, 96, 77, 104, 106, 129, 100, 36, 52, 31, 128, 109, 118, 105, 71, 124, 30, 58, 21, 19, 81, 6, 91, 101, 57, 16, 89, 94, 24, 42, 116, 3, 70, 122, 15, 2, 90, 49, 84, 55, 72, 86, 63, 114, 7, 46, 50, 41, 68, 79, 32, 130, 132, 121, 0, 54, 12, 33, 111, 18, 103, 11, 27, 37, 95, 125, 45, 28, 74, 66, 78, 48, 61, 83]

For size 134, the time was 1.6291337200000022.

[59, 105, 23, 124, 62, 35, 47, 69, 48, 52, 110, 16, 101, 24, 32, 79, 63, 80, 46, 70, 1, 126, 120, 0, 134, 109, 78, 42, 11, 83, 75, 50, 125, 77, 29, 9, 103, 21, 95, 113, 127, 89, 38, 60, 100, 27, 8, 41, 2, 117, 5, 129, 133, 106, 33, 22, 93, 91, 7, 56, 132, 61, 13, 51, 3, 130, 65, 87, 34, 40, 102, 98, 107, 82, 64, 53, 74, 18, 14, 39, 104, 26, 55, 68, 94, 71, 12, 6, 28, 19, 121, 84, 118, 97, 17, 131, 108, 43, 111, 88, 123, 81, 72, 128, 67, 20, 122, 25, 99, 36, 66, 114, 49, 57, 116, 76, 85, 45, 119, 44, 112, 15, 31, 58, 96, 4, 90, 92, 115, 86, 73, 37, 54, 10, 30]

For size 135, the time was 0.24659760599999814.

[44, 121, 109, 81, 77, 40, 9, 71, 51, 97, 32, 117, 92, 42, 98, 49, 63, 133, 120, 52, 82, 10, 101, 134, 62, 124, 31, 90, 18, 126, 3, 84, 58, 41, 85, 129, 47, 108, 15, 26, 57, 113, 30, 100, 59, 95, 64, 1, 96, 72, 70, 119, 61, 46, 39, 11, 13, 89, 114, 118, 76, 115, 6, 132, 110, 102, 68, 22, 135, 130, 122, 66, 8, 0, 116, 43, 125, 56, 4, 45, 27, 80, 103, 123, 16, 23, 87, 54, 17, 20, 131, 91, 2, 21, 7, 65, 60, 34, 73, 48, 128, 24, 78, 75, 111, 83, 112, 55, 127, 74, 53, 5, 93, 35, 38, 107, 19, 99, 14, 88, 106, 104, 67, 86, 29, 33, 25, 69, 37, 50, 12, 28, 36, 79, 105, 94]

For size 136, the time was 0.25233678799999737.

[12, 60, 132, 90, 97, 4, 123, 82, 131, 22, 128, 27, 88, 14, 136, 79, 43, 129, 64, 16, 127, 95, 8, 126, 112, 51, 66, 63, 17, 21, 92, 113, 99, 15, 114, 133, 5, 71, 29, 80, 55, 47, 110, 36, 24, 41, 10, 7, 9, 104, 67, 54, 124, 130, 72, 134, 40, 23, 116, 98, 58, 3, 1, 74, 68, 96, 103, 18, 111, 56, 44, 119, 2, 38, 28, 37, 106, 50, 13, 121, 87, 135, 25, 20, 59, 107, 100, 33, 45, 91, 31, 49, 101, 11, 84, 89, 46, 65, 0, 76, 70, 120, 57, 6, 19, 125, 39, 78, 52, 117, 86, 94, 122, 53, 93, 26, 83, 102, 118, 35, 115, 73, 81, 42, 34, 48, 75, 105, 109, 85, 61, 32, 77, 62, 30, 69, 108]

For size 137, the time was 0.35411206999999933.

[77, 24, 137, 5, 74, 52, 48, 117, 96, 110, 84, 54, 58, 131, 19, 44, 37, 111, 32, 112, 9, 62, 16, 98, 109, 38, 45, 125, 123, 20, 93, 49, 71, 34, 134, 55, 60, 65, 94, 13, 136, 7, 128, 25, 56, 114, 103, 22, 31, 12, 105, 102, 88, 33, 115, 26, 29, 97, 2, 119, 90, 51, 47, 108, 50, 130, 35, 100, 78, 133, 87, 66, 3, 135, 39, 73, 8, 81, 116, 43, 107, 53, 82, 23, 41, 129, 95, 0, 30, 27, 17, 126, 46, 69, 14, 101, 127, 104, 85, 80, 99, 89, 63, 106, 1, 42, 132, 122, 18, 68, 72, 4, 61, 121, 92, 83, 40, 113, 57, 67, 10, 15, 21, 6, 76, 118, 86, 124, 64, 75, 36, 11, 79, 28, 59, 91, 120, 70]

For size 138, the time was 0.27519125899999963.

[11, 9, 101, 129, 68, 114, 122, 67, 40, 128, 106, 52, 27, 2, 35, 74, 8, 47, 136, 90, 3, 31, 88, 98, 137, 41, 91, 64, 13, 97, 43, 15, 93, 120, 112, 32, 108, 92, 45, 53, 0, 14, 123, 7, 37, 57, 25, 48, 6, 10, 124, 71, 75, 102, 130, 60, 21, 119, 121, 111, 117, 29, 89, 49, 59, 34, 69, 118, 50, 87, 96, 5, 16, 79, 62, 23, 30, 105, 18, 70, 78, 26, 126, 55, 84, 72, 1, 58, 20, 134, 138, 110, 125, 127, 36, 76, 100, 132, 65, 116, 94, 12, 133, 17, 82, 42, 44, 131, 61, 66, 85, 113, 28, 109, 77, 19, 86, 83, 24, 22, 4, 95, 81, 103, 73, 115, 46, 54, 104, 38, 107, 33, 56, 80, 39, 63, 135, 99, 51]

For size 139, the time was 949.467909451.

Now, I am working on two different implementations that I don’t think I will figure out before the deadline.

First, I am exploring variations in getting the most constrained. Instead of simply calculating the row with the smallest number of possible columns, I would calculate a list of the most constrain**ing**. The difference is that the most constraining method would generate an ordered list of the row and column that, if a queen was placed there, would introduce the most constraints on the board. So far, finding an efficient way to implement this code had been wracking my brain. I have thought of using a heap to store the ordered spaces, however the way I calculate the order is at least O(n^5) so it is not practical to implement into my csp yet.

Second, I did some research on algorithms that could solve immensely bigger sizes in less than a second. I came across two algorithms: hillclimber (iterative, not recursive so does not apply as a “csp” improvement) and QS4. QS4, at least how I understood the reports, is premised on the fact the most time consumed is in the backtracking/recursive process. QS4 would limit this by placing the first n rows without the queens attacking each other and then randomizing the rest. To find the correct path, the program would swap column values until the number of “collisions” would be minimized. There are a few problems. I am having a hard time implementing an efficient compute\_collisions method to even begin the swapping portion of QS4. However, more complicated, is how and when to do the actual swapping. If I reduced collisions immediately after I finished filling up the list, I would essentially be doing calculating the solution completely iteratively. However, if I reduce collisions after I fill my list as the recursion travels back up each level, I would not be able to guarantee the correct path every time. I am still working on these implementations, but I doubt I will be able to find the solution soon.